Metric Convolutions:

A Unifying Theory to Adaptive Image Convolutions

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https://github.com/Tommoo/MetricConvolutions

Convolution zoo

Standard convolution

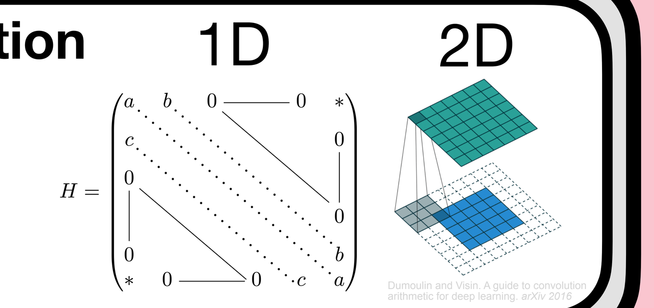
$$(f \star h)(x) = \int_\Omega f(x+u)\,h(u)du$$

- Linear • Local averaging • Kernel weight sharing
- Sliding kernel window (Changing) • Shift-equivariant
- Signal independent

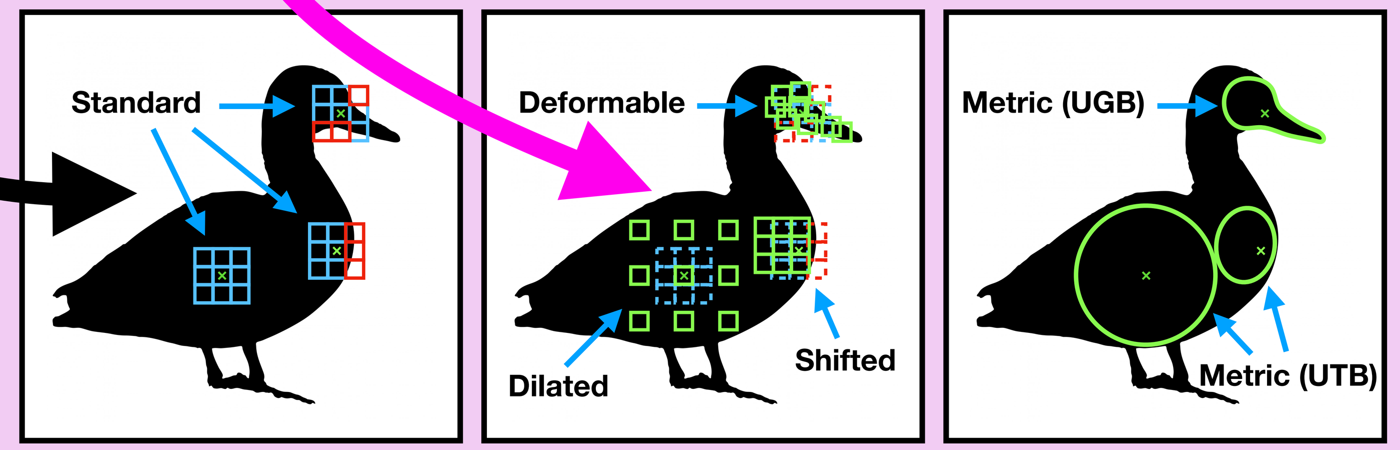
Discretisation 1D 2D

$k \times k$ grid

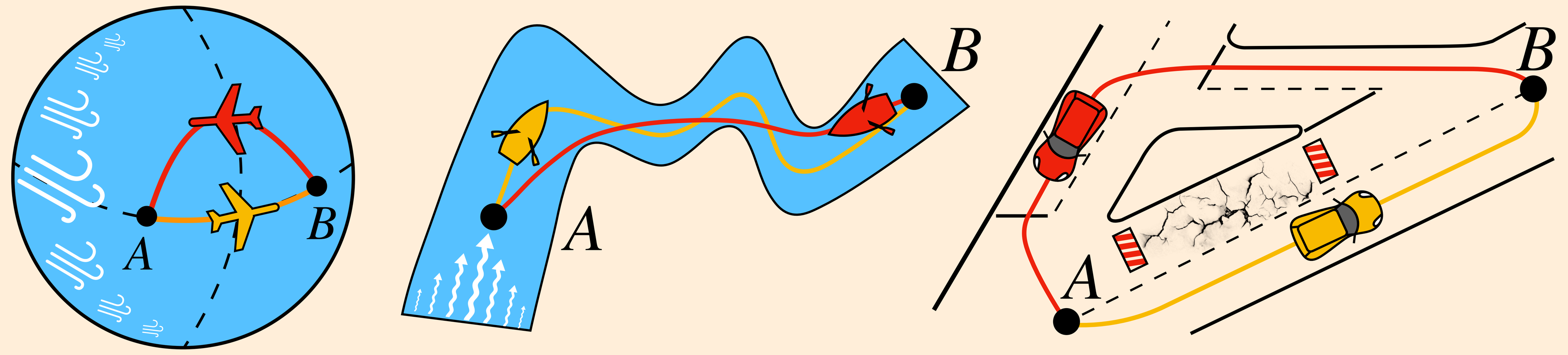
3×3



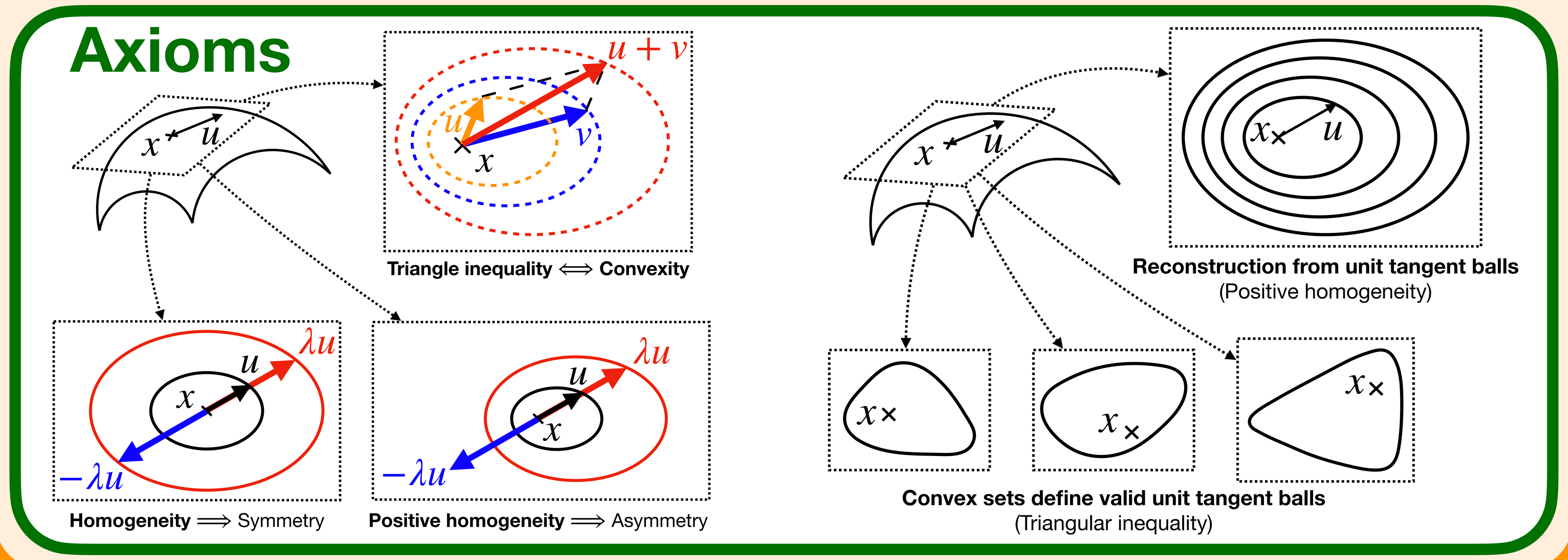
Adaptive convolutions



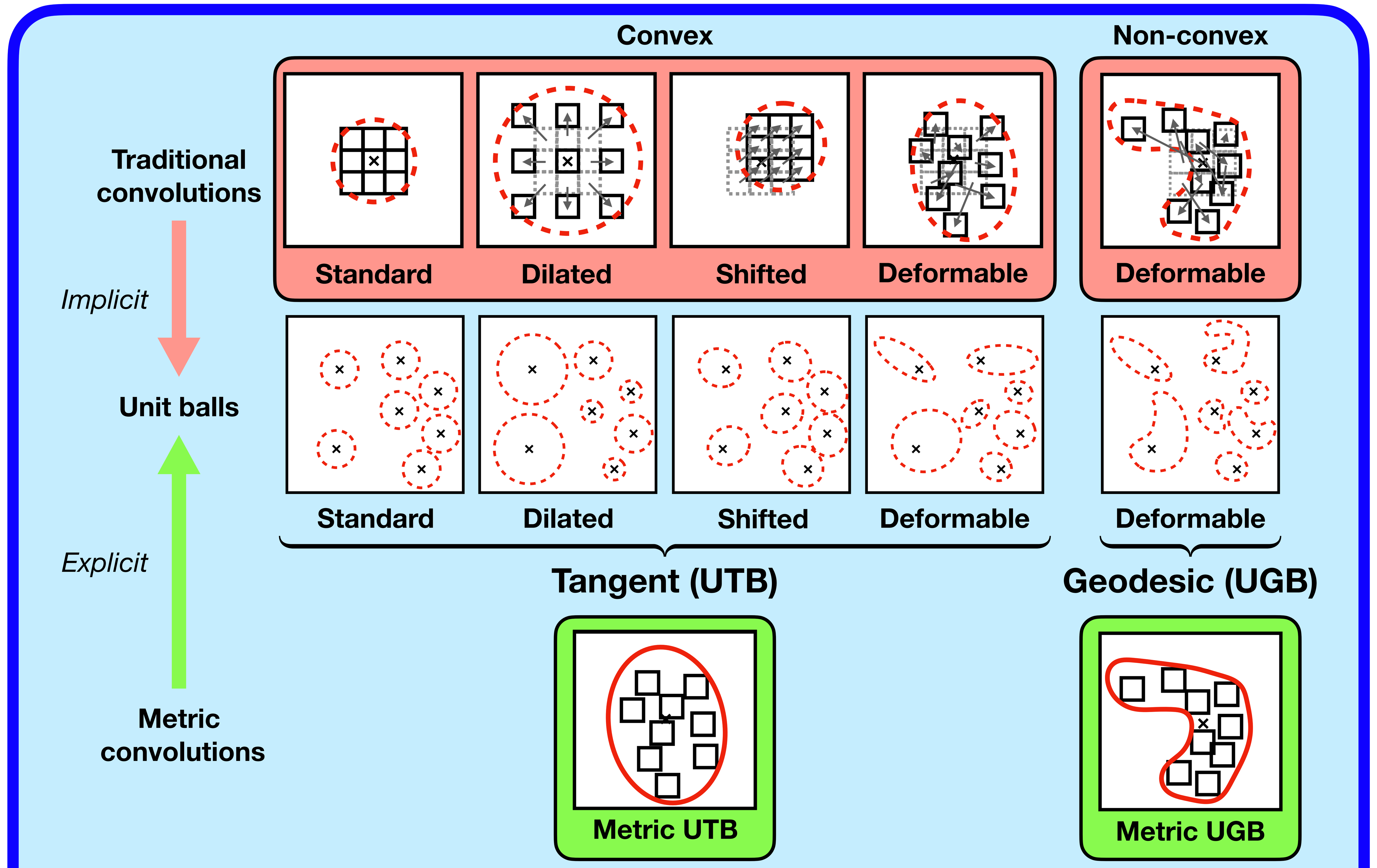
Unit balls in Finsler geometry



Axioms



Unifying theory



Theorem: Convolutions implicitly sample unit balls of some (a)symmetric metric

$$(f \star g)(x) = \int_{\Delta_x} f(x+y)g(y)dm_x(y)$$

In practice:

Parametric metrics → Analytical unit balls (tangent)

Riemann

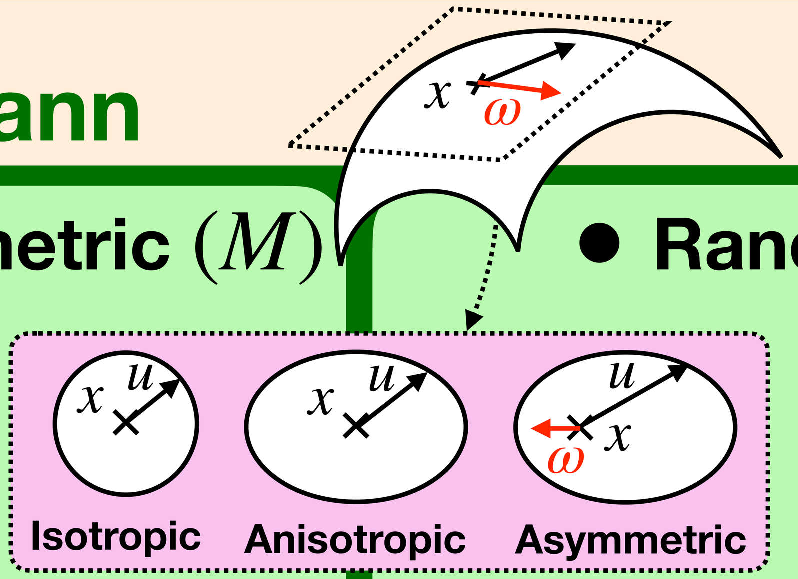
- **Riemann metric (M)**

$$R_x(u) = \|u\|_{M(x)}$$
$$R_x(u) = R_x(-u)$$
$$\downarrow$$
$$d_R(A,B) = d_R(B,A)$$

Finsler

- **Randers metric (M, ω)**

$$F_x(u) = \|u\|_{M(x)} + \omega(x)^\top u$$
$$F_x(u) \neq F_x(-u)$$
$$\downarrow$$
$$d_F(A,B) \neq d_F(B,A)$$



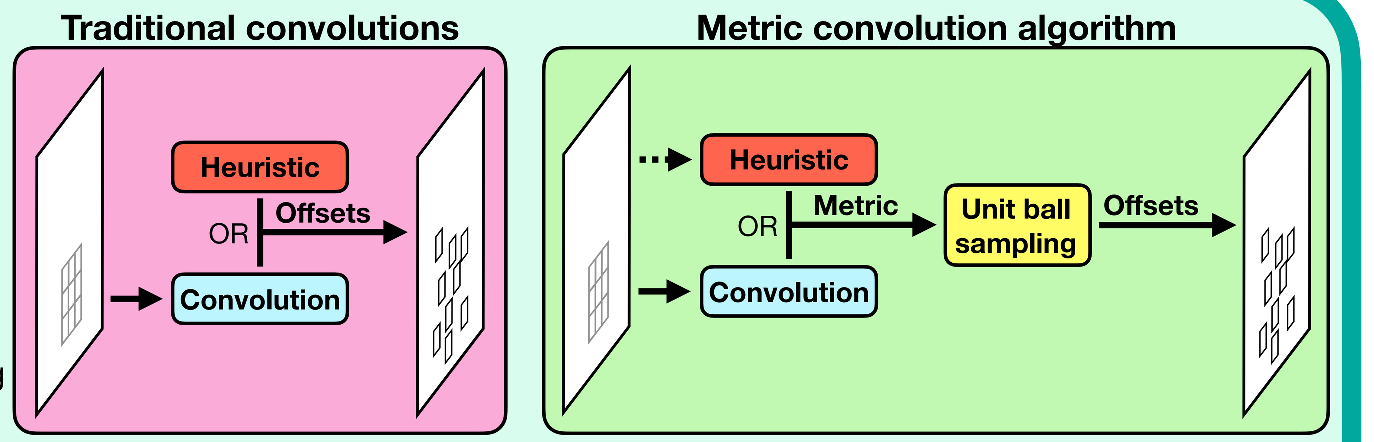
Metric convolutions: Explicit metrics

Pipelines

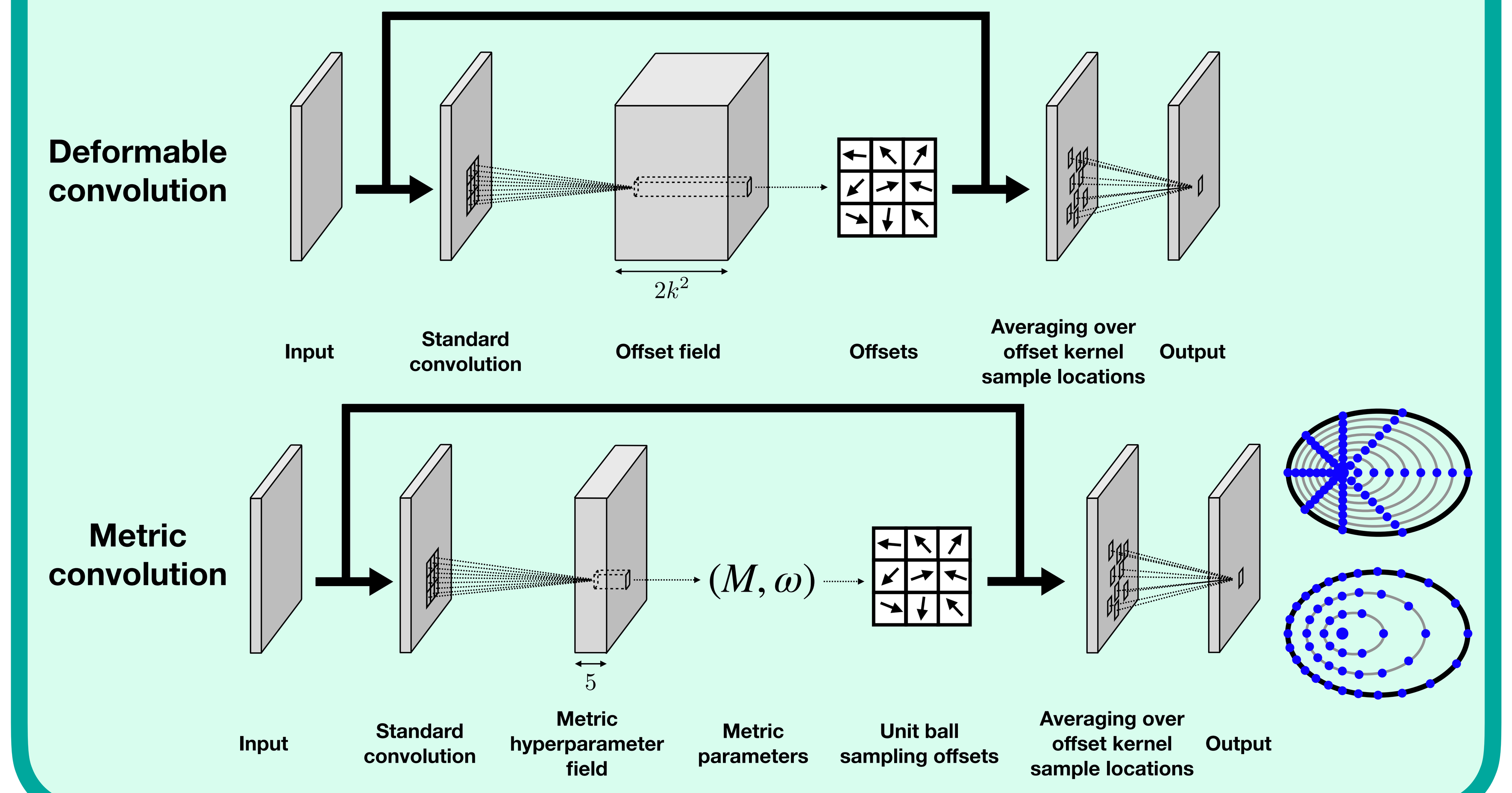
- Geometric heuristics

AND / OR

- Differentiable learning



Implementation



Results